Problem 0.

(a) Read the syllabus and familiarize yourself with the course website (especially the schedule).

(b) Log into Google using your Drake credentials, and then complete the Background Questionnaire for the course.

(c) Do the first reading (Sipser §§0.1–0.2) and complete your first reading journal. Reading journals are due at 8:00 AM the morning of the corresponding day on the course schedule, so this journal is due on Thursday, January 30th at 8:00 AM. Please ensure the subject of your email is:

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[CS 139] Reading Journal: Sipser 0.1-0.2
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(d) Finish setting up your Gradescope account. You should have received an email with the title "Welcome to Gradescope for CS 139" asking you to set your password. You will be submitting all of your assignments on Gradescope (including this one).

Problem 1. Recall that we use $\mathbb{N} = \{1, 2, 3, \ldots\}$ to denote the set of natural numbers. For two natural numbers $a, b \in \mathbb{N}$, we say that a *divides* b, and we write $a|b$, if b can be divided by a with no remainder. If $a|b$, we also say that b is *divisible* by a.

For $a \in \mathbb{N}$, we also define the set

$$a|\mathbb{N} = \{b \in \mathbb{N} \mid a \text{ divides } b\}.$$

(a) Write out the first 5 elements of the set $7|\mathbb{N}$

(b) Determine whether or not the statement $8|\mathbb{N} \subseteq 2|\mathbb{N}$ is true or false. Explain your answer.

Problem 2. Recall that a function $f : A \to B$ is *one-to-one* (i.e. injective) if and only if the following holds: for all $x, y \in A$, if $f(x) = f(y)$, then $x = y$. We also say that f is *onto* (i.e. surjective) if and only if the following holds: for all $y \in B$, there exists an $x \in A$ such that $f(x) = y$.

Give an example of a function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ that is onto but is not one-to-one. Explain your answer.

Problem 3. Let $\mathbb{N}$ be the *universe* set for this problem. This means that the *set complement* operator is defined as

$$\overline{X} = \mathbb{N} \setminus X = \{x \in \mathbb{N} \mid x \notin X\},$$

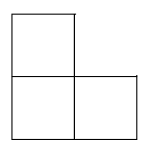
for all sets $X \subseteq \mathbb{N}$. For example, given the set $E = \{2, 4, 6, 8, \ldots\}$, the complement of E is the set $\overline{E} = \{1, 3, 5, 7, \ldots\}$.

Determine whether or not the following statement is true or false.

$$\text{If } A \subseteq \mathbb{N} \text{ and } B \subseteq \mathbb{N}\text{, then } \overline{A \cup B} = \overline{A} \cup \overline{B}$$

Explain your answer.

Bonus Problem (Extra Credit)**.** Dr. Adam Case challenges you to a game. He gives you a board upon which is drawn a grid of squares with 2^n rows and 2^n columns (i.e. there are $(2^n)^2$ squares in the grid). He also gives you a bag containing a large number of L-shaped pieces like the one shown below. (Each square is the size of a space on the grid.)



The challenge is this: Adam will first select a single square on the grid and mark it as unusable. If you can then place the L-shaped tiles in such a way that they cover the entire grid (with no overlaps and leaving only the unusable square uncovered), you win. Otherwise, Adam wins.

Prove that you can win the game for any n and for any unusable square that Adam selects. (A proof by induction is the easiest approach.)