

Problem 0. Document how much time you spend on each of the following problems and cite any resources you received help from.

Problem 1. We know that P and NP are classes of decidable languages. We also know that decidable languages are closed under union, intersection, and concatenation. What about P and NP ?

- (a) For each of operation above, argue whether or not P is closed under that operation.
- (b) For each of operation above, argue whether or not NP is closed under that operation.

Problem 2. A *simple path from x to y* in an undirected graph $G = (V, E)$ is a sequence of adjacent vertices that begins with x , ends with y , and which no intermediate vertex appears twice. Define the language

$$\text{PATH}_{\text{SHORT}} = \{\langle G, x, y, k \rangle \mid G \text{ is an undirected graph and contains a simple path from } x \text{ to } y \text{ of length at most } k\}.$$

Prove that $\text{PATH}_{\text{SHORT}}$ is in the class P .

Bonus Problem (Extra Credit). Prove that the language

$$\text{NOPAL}_{\text{DFA}} = \{\langle D \rangle \mid D \text{ is a DFA that does not accept any palindrome}\}$$

is decidable.

Bonus Problem (Extra Credit). Let L be the language defined by

$$L = \{\langle M, w \rangle \mid M \text{ is a TM that on input } w \text{ never overwrites any symbol of } w \text{ during its computation.}\}$$

Prove or disprove that L is decidable.